AN OPTIMAL INVENTORY CONTROL OF RAW MATERIALS AND NETWORK ANALYSIS OF PRODUCTION PLANNING FOR AKA PAINT INDUSTRY NIGERIA LIMITED

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ABSTRACT

This study analyzed an optimal inventory control of raw materials and network analysis of production planning for Aka Paint Industry Nigeria Limited. Inventory and network analysis data from a secondary source, obtained from Aka Paint Industry Nigeria Limited was utilized by the study. Mathematical models such as Economic Order Quantity (EOQ) model were used in determine the optimal quantity of raw materials to be kept on inventory. Programme Evaluation and Review Techniques (PERT) and Critical Path Method (CPM) were used in estimating the completion time and expected duration respectively. The Optimum quantities for the various raw materials were calculated with their inventory length. Also the minimum and the expected duration of the production were obtained. An optimum quantity of 12,409 buckets of water, 9,475 bags of dolomite and 70,372kg of chemicals were obtained, and their inventory lengths were found to be one year for each item. Besides, a total completion time for 59 days and expected duration of 84 days of the production were obtained. In conclusion, it is recommended that the Company should adopts EOQ model in knowing the optimal quantity of raw materials to be ordered for and when best to make the order, and also use PERT and CPM in knowing the optimal scheduling activities.

***Keywords**:* Inventory, network planning, paint production, EOQ, PERT, CPM

INTRODUCTION

In production, the development of any firm depends on the growth and utilization of raw materials. Production planning is one of the most relevant fields of study in operation research and management science. The increased techniques and advanced in technology, organizations ensure steady production and supply of their goods by storing certain quantities of raw materials as buffer against unprecedented short fall in their supplies. Certain stocks of finishing goods are maintained in order to meet up with unforeseen demand or short -notice orders. The best planned and managed enterprises survive and efficiency of a company's operation is directly related to the inventory situation existing within it.

Inventories are the soul and life of any manufacturing organization. Inventory control enables organization to determine when it is necessary to stock physical goods or commodities for the purpose of specified periods of time. Amount of inventory results to high capital costs, high operating cost and increased production efficiency when too much space is used while insufficient inventory leads to inability of the firm to meet up shut downs if there occurs drop or half in supply of raw materials. (Inyama and Osuagwu 1999).

Network analysis is a family of related techniques developed to aid management in planning and control of projects scheduling and coordinating various inter- related activities that define a project. In a production of specified duration, network analysis is inter-relationship of the various jobs, or task which make up the overall and obvious identity the critical path or the production. (Sharma J.K 1997)

Materials and Methods

In this study, we shall apply only two methods of network analysis; these include Critical Path Method (CPM) and Programme Evaluation and review Technique (PERT). Programme Evaluation and Review Technique is mostly used in research development that is generally not known before-hand. Therefore uncertainty in activity duration is a random variable, in order to forecast accurately the mean duration of each project activity and project duration, PERT is used. It enables us to determine the probabilities of finishing various stage of project at a given time. The three programme evaluation and review techniques (PERT) model estimates for each completion time of each activity in the project network are the most probable (likely) time, the optimistic time and the pessimistic time. (Sharma 2010)

Method of data collection:

Data collected usually, in operations research studies is the business of an operations research team. The data for this study are secondary data of the raw materials and network analysis used in the production of paint at Aka Paint Nigeria limited. The data were collected differently for inventory control of raw materials and network analysis of the production planning.

Raw material used:

The raw materials uses in the production of paint are ultra-chinese, kaolin, titanium, kerosene, resin, genefour, ammonia, anticide, bermociol, and defoamer). They are classified into three raw materials namely; Water, Dolomite and Chemicals.

Table 1. For raw materials

Raw Materials	¤ Water	Dolomite	Chemicals
Quantity ordered per cycle (Q)	43 buckets daily	26 bags daily	192.8 kg daily
Demand rate (D)	43 buckets daily	26 bags daily	193.3 kg daily
Lead time (L)	1 daily	I daily	1 daily
Holding cost per item (C_h)	#28.00 per bucket	#198.00 per bag	#245.00 per kg
Order/Set up cost (C_O)	#35.00 per bucket	#199.00 per bag	#213.00 per kg

Source: Aka paint Nigeria Limited

2.3 Inventory Model: The Economic Order Quantity (EOQ)

The Economic Order Quantity (EOQ) is defined as the ordering quantities which minimize the balance of cost between inventory holding costs and re-order cost. (Lucey 1992). In this model, we used single item static model (shortage not allowed) this model is called basic inventory model or economic lot size model. Here, the objective of selecting an inventory policy, that is to choose an economic Order quantity (EOQ) the ordering frequency (time when an order must be placed) the purpose of using EOQ model in this research is to find out the particular quantity which minimize yearly inventory cost, that is the total ordering and carrying costs. The Economic order quantity is the best known and most fundamental inventory decision model.

Assumptions of Economic Order Quantity (EOQ)

1. That there is a known constant, ordering cost
2. That there is a known, constant stock holding costs
3. That the rate of demand are known
4. That there is a known constant price per unit
5. That replenishment is made instantaneously, i.e the whole batch is derived at once.
6. No stock outs are allowed.
7. Lead time has to be zero or at least constant and known

Equation of an Economic Order Quantity Model

The EOQ formula is given below, it's derivation

Total variable cost of inventory (T_c) = $DC + \frac{DC_o}{Q} + \frac{C_h Q}{2}$

The optimum value of Q is obtained by minimizing T_c with respect to Q, we have

$$\frac{\sigma T_c}{\sigma Q} = \frac{-DC_0}{Q^2} + \frac{C_h}{2}$$

Equating the derivative to zero, we have

$$\frac{-DC_0}{Q^2} + \frac{C_h}{2} = 0$$

Cross multiply, we have

$$C_h Q^2 = 2DC_0$$

Making Q^2 the subject of the formula, we have

$$Q^2 = \frac{2DC_0}{C_h}$$

Therefore $$Q^* = \sqrt{\frac{2DC_0}{C_h}} = EOQ$$

Where,

D is denoted by annual demand

C_o is denoted by ordering/setup quantity cost per order

C_h is denoted by holding/carrying cost per unit stock

Q is denoted by optimal order quantity/ EOQ

To get the optimum inventory cycle length i.e the time it takes to withdraw optimum value of Q^* is t $$t_0 = \frac{1}{D}\sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2C_0}{C_h D}}$$

therefore $$t_0\ n = \frac{2C_0}{C_h D}$$

Network analysis of production planning

In this study, the list of the activities was compiled and the activity duration, as well as the labour requirement was later determined. Having identified all the activities, the precedence and interdependences relationships among the activities was determined. These were arrived by finding for each activity and its immediate predecessor, after which the facts were re-arranged for modeling. Due to uncertainty in the duration of the production activities, and in order to account for each uncertainty.

Network analysis model

In this study, we shall apply only two methods. These include CPM and PERT which are used in network analysis of the production planning.

Critical Path Method (CPM)

Critical Path Method (CPM) does not involve probability but actual time estimate of activities in a project.

The critical path method (CPM) is used to determine the following:

1. The earliest possible start time for activities is denoted by
   $$T_j(E) = Max\left[T_i\ (E) + t_{i_j}\right]$$
2. The latest allowable time for the activities
   $$T_i\ (L) = \begin{cases} Min\ \ T_j\ (L) - t_{i_j} \\ T_n(E)\ for 1 = n \end{cases}$$
3. The critical path

The critical path is the sequence of activities that form a continuous path between the start of the project and its completion. The critical path is shown by a thick line or double lines in the network diagram. The length of the critical path is the sum of the individual times of all critical activities lying on it and defines the longest time to complete the project. The critical path identified all the critical activities of the project, before determining the critical path, the slack times for the event is as follows:

$$S_i = T_i(L) - T_j(E),$$

For each activity is first determined the events, which have zero slack times are on the critical path.

Programme Evaluation and Review Techniques (PERT)

The information from the manager on the time estimates of the company, enables them to develop and measure of completion time for a project. To measure of dispersion, the mean and standard deviation of the completion time distribution, the probability of finishing the project at a given due date can be estimated. PERT is a probability time estimate for activities in a project.

Assumptions of Programme Evaluation and Review Techniques (PERT)

In PERT, there are three-time estimates for each activity, assuming a beta probability distribution for the time estimates. These estimates are designated as:

1. **The most probable (likely) time**: This the time required to complete an activity under normal condition and is denoted by (m).
2. **The Optimistic time**: this is the shortest possible time (duration) in which an activity can be performed assuming that everything goes well, and is denoted by (a)
3. **Pessimistic tim**e: This is the longest time required to perform an activity under extremely bad condition; like NEPA blackout, mechanical breakdown, and workers strike delay in delivery of goods and services, and is denoted by (b).

For uncertainty:

$$\mu = \frac{a + 4m + b}{6} = mean$$

$$\sigma^2 = \left(\frac{b - a}{6}\right)^2 = Variance$$

$$\sigma = \left(\frac{b - a}{6}\right) =$$ Standard Deviation

The expected length (duration) of the project

$\sum(T) =$ sum of the average times of the work

The probability of meeting specific production dead line can be calculated using the standard score and the area under the normal curve table.

$$Z = \frac{T - \Sigma(T)}{\sigma(T)}$$

Where,

T is denoted by the required days

$\sum(T)$ is denoted by sum of the average time

$\alpha(T)$ is denoted by standard deviation of the production

Table 2: Activities of the production of paint and there different duration.

Job	Activity	Duration days	Immediate predecessor
A	Reception of raw materials	30	—
B	Laboratory test (chem.)	2	A
C	Storage	6	B
D	Mixing	2	B,C
E	Fabrication and physical test	3	D
F	Profile and making and die run	2	D
G	Profile and fabric cutting	3	E,F
H	Light and heavy weight making	4	G
I	Verification before cure	2	H
J	Curing	1	I
K	Verification after cure	1	J
L	Marketing	7	K

Source: Aka paint Nigeria Limited

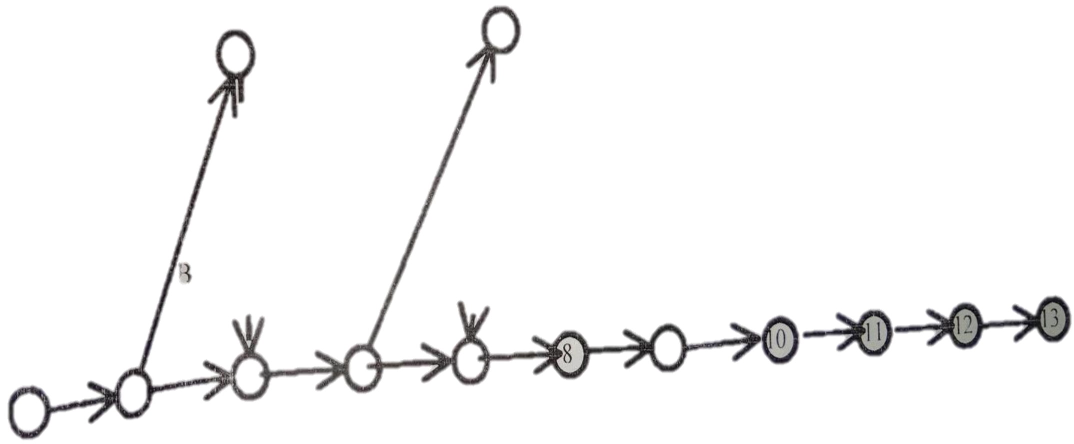


Fig. 1: Network Diagram

Tables 3 and 4 display data generated through the different activities of the production of paint and there different duration.

Table 3: Activities of the production of paint and there different duration.

Job	Immediate predecessor	Most likely time (m)	Optimistic time (a)	Pessimistic time (b)
A	None	30	45	90
B	A	2	5	7
C	B	6	8	10
D	B, C	2	4	8
E	D	3	6	8
F	D	2	4	6
G	E, F	3	5	7
H	G	4	7	11
I	H	2	7	8
J	I	1	3	5
K	J	1	3	6
L	K	7	13	16

Results and discussions

Calculation of Optimal quantity of raw materials

Using table 1: By applying the EOQ the optimum quantity for the various raw materials to be ordered and when best to make the order are calculated.

For Water:

Let Q_w be the optimum quality of water to be ordered for and t_w be the inventory length

$$Q_w = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times 43\times 365\times 35\times 43\times 365}{28}} \quad = \quad 24{,}816 \text{ buckets}$$

Inventory length

$$T_w = \frac{24816}{43\times 365} = 1.581 \cong 2 \ years$$

In one year they should order = $\frac{24816}{2} = 12408 \, buckets$

For Dolomite:

$$Q_d = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times 26\times 365\times 199\times 36\times 365}{198}} = 13455 \, bags$$

Inventory length

$$T_d = \frac{Q_d}{D} = \frac{13455}{26\times 365} = 1.42 \ year$$

D =

In one year they should order

$$= \frac{13455}{1.42} = 9475\,bags$$

For Chemicals:

$$T_c = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times193\times365\times213\times193\times365}{245}} = 92891kg$$

Inventory length

$$T_c = \frac{Q_c^*}{D} = \frac{92891}{193\times365} = 1.32\ year$$

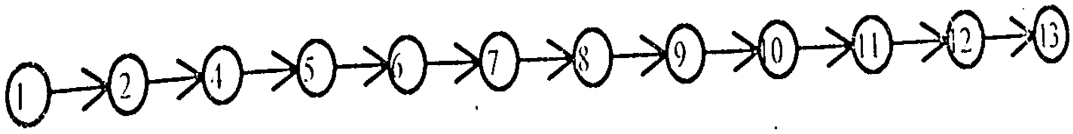
In one year they should order $\frac{92891}{1.32} = 70372\,kg$

Calculation of minimum and expected duration in network analysis:
The table below, shows the latest and earliest time of each event and also its slack times, which helps to identify the Critical path.

Table 4: The latest, earliest, and slack times of the event

Events	T_j (L)	T_i (E)	S = T_j (L) – T_i (E)	Remark
1	0	0	0	Critical
2	30	30	0	Critical
3	32	36	4	Non critical
4	36	36	0	Critical
5	38	38	0	Critical
6	41	41	0	Critical
7	41	41	0	Critical
8	44	44	0	Critical
9	48	48	0	Critical
10	50	50	0	Critical
11	51	51	0	Critical
12	52	52	0	Critical
13	59	59	0	Critical

Fig. 2: the critical path is:



Expected duration of the production and its variance

The expected duration $\sum(T)$ = sum of the average time of the job in the critical path are A,B,D,E,F,G,H,I,J,K and L.

Table 5: Shows the average time of jobs and their variances

Jobs	Immediate predecessor	m	a	b	μ	σ^2
A	None	30	45	90	43	56 ¼
B	A	2	5	7	3	1/9
C	B	6	8	10	7	1/9
D	B, C	2	4	8	3	4/9
E	D	3	6	8	4	1/9
F	D	2	4	6	3	1/9
G	E, F	3	5	7	4	1/9
H	G	4	7	11	6	4/9
I	H	2	5	8	4	1/9
J	I	1	3	5	2	1/9
K	J	1	3	6	2	1/9
L	K	7	13	16	10	1/9
TOTAL					84	58.56

$\sum(T)$ = sum of the average time of the job in the critical path are
A,B,D,E,F,G,H,I,J,K and L.
= (43+3+3+4+3+4+6+4+2+2+10) = 84 days

σ^2(T) = sum of variances of jobs A,B,D,E,F,G,H,I,J, and K

$$56\frac{1}{4}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{4}+\frac{1}{9}+\frac{1}{4}+\frac{1}{4}=\frac{2108}{36}=58.56$$

The Standard deviation of the production deviation = $\sqrt{58.56} = 7.65$

The probabilities of the assumption that the production will be completed in less or more days than the expected duration are calculated for effective planning.

The probabilities of completing the production in:

1 Less than 81 days
2 More than 90 days
3 Less than 86 days

$$Z = \frac{T-\sum(T)}{\sigma(T)}$$

Where,

T is denoted by the required days
$\sum(T)$ is denoted by sum of the average time
$\alpha(T)$ is denoted by standard deviation of the production

1. The probability of less than 81 day = P (T < 81)
   Convert 81 to s-tandard score
   $$Z_1 = \frac{81-84}{7.64} = -0.39$$
   Z = 0.5 – (area between Z = 0 and - 0.39) = 0.5 – 0.1517 = 0.35
   There about 35 chances in hundred that production will be completed in less than 81 days

2. The probability of more than 90 day = P (T > 90)
   Convert 90 to standard score
   $$Z_2 = \frac{90-84}{7.64} = 0.78$$
   Z = 0.5 – (area between Z = 0 and 0.78) = 0.5 – 0.2823 = 0.22
   There about 22 chances in hundred that the production will be completed in more than 90 day.

3. The probability Less than 86 days = P (T < 86)
   Convert 86 to standard score
   $$Z_3 = \frac{86-84}{7.64} = 0.26$$
   Z = 0.5 + (area between Z = 0 and 0.26) = 0.5 + 0.1026 = 0.60
   There about 60 chances in hundred that the production will be completed in less than 86 days.

CONCLUSION

At this point, it is clear that to calculate the optimal quantity of raw materials used in the production of paint in aka paint Industry limited, in order to know the quantities to be ordered for and when best to make the order.

In the frame of this research, data analysis, estimation, network of the different activities, the minimum and expected duration of the production were calculated using PERT and CPM. The result shows that optimum quantity of 12,409 buckets 9,475 bags and 70,372 kg were obtained from water, dolomite and chemicals respectively and their inventory length were found to be one year for each item. The production planning of the company was revealed that completion time and expected duration are 59 days and 84 days respectively. The chance of completing the production earlier them expected duration was calculated, it was obtained that there is little or no chance of completing the production earlier than expected duration.

This implies that production planning of any paint industries that have the same system of operation like Aka paint industry limited will find this research work very useful as guide to achieve better with whatever is available.

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